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Adaptive guided image filter for warping in variational optical flow computation

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ABSTRACT

The variational optical flow method is considered to be the standard method to calculate an accurate dense motion field between successive frames. It assumes that the energy function has spatiotemporal continuities and appearance motions are small. However, for real image sequences, the temporal continuity assumption is often violated due to outliers and occlusions, causing inaccurate flow vectors at these regions. After each warping operation, errors are generated at the corresponding regions of the warped interpolation image. This results in an inaccurate discrete approximation of the temporal derivative and thus ends up affecting the accuracy of the estimated flow field. In this paper, we propose an adaptive guided image filter to correct these errors in the warped interpolation image. A guidance image is reconstructed by considering both the feature of the reference image as well as the difference between the warped interpolation image and the reference image, to guide the filtering of the warped interpolation image and the reference image, to guide the filtering of the warped interpolation image and confidence measure. Extensive experiments on different datasets and comparison with state-of-the-art variational optical flow algorithms demonstrate the effectiveness of our method.

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1. Introduction

Estimation of the apparent motion of a scene plays a fundamental role in computer vision and image processing. Currently, one of the most successful techniques that addresses this problem is the variational optical flow method [1,2], which formulates an energy function by aggregating a data term and a smoothness term. Minimizing the energy function can get the motion of each pixel. Technically speaking, the minimization involves solving the corresponding Euler–Lagrange equations of the energy function numerically [3]. The calculated flow field is a dense field of displacement vectors, which describes how corresponding pixels in consecutive frames match. The direction and magnitude of each flow vector indicate where and how far a pixel moved. This property ensures the optical flow method is widely used in, for example, object tracking [4] and segmentation [5], super-resolution reconstruction [6], and medical diagnostics [7].

The variational optical flow method proposed by Horn and Schunck (HS) [8] is based on two assumptions: 1) the *brightness*

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http://dx.doi.org/10.1016/j.sigpro.2016.02.018 0165-1684/© 2016 Elsevier B.V. All rights reserved. constancy assumption (BCA), which assumes that the brightness of a pixel does not change along its motion trajectory over time; and 2) the smoothness assumption, which assumes that the flow field is smooth and ensures the optical flow problem is well posed. In practice, these two assumptions are rarely satisfied. Especially under challenging conditions, such as outliers (e.g., image noise and estimated flow errors [9]), large displacements, occlusions and illumination changes, the performance of HS is notably worse [40]. Furthermore, the HS model is too slow to be implemented in realtime as the large and sparse partial differential equations (PDEs) is computationally expensive. Fortunately, in the past 30 years, considerable progress has been achieved. The progress can be divided into two classes: 1) different advanced concepts are integrated into the variational framework, to enable the methods to preserve discontinuity [5,10,11], to handle large displacements [1,2,20], to treat illumination changes [12,13], to be robust with respect to outliers [9,15], and to tackle occlusions [9,16,17]; and 2) efficient and advanced optimization schemes are presented to target real-time use [3,18].

The popular variational algorithms are focused on how to improve the data term (Eq. (1)) by incorporating new assumptions [19], or to design advanced smoothness constraints that preserve discontinuity. There are few literatures concern the basic feature of the numerical computation, where the flow field is computed from





SIGNAL PROCESSING the spatiotemporal derivatives of image intensities. Therefore, the input images and the intermediate warped interpolation image I_{warp} (I_{warp} refers to the warped interpolation image based on the flow field calculated in the previous step, see Eq. (2)) are constrained to be continuous and differentiable in space-time. Simultaneously, the flow vector should be small and valid at every pixel. Errors are typically produced in the intermediate flow fields due to outliers and occlusions. Subsequently, I_{warp} will contain errors caused by these inaccurate flow vectors. Furthermore, the spatiotemporal derivatives will be distorted. Consequently, the accuracy of the final estimated flow fields and the I_{warp} during optimization is a good way to improve the performance of the variational algorithms.

Till now, the majority of variational optical flow algorithms try to reduce errors during the numerical computation, concentrating on applying different filters to smooth the intermediate flow fields to remove outliers or to correct flow errors by integrating useful information [10,21].

Xiao et al. [17] proposed a multi-cue driven adaptive bilateral filter (BF), which enables smoothing of the flow field with highly desirable motion discontinuity preservation. The algorithm can handle occlusions partially, but, BF is time consuming. Wedel et al. [12] introduced a median filter (MF) to denoise the flow field, but the MF over-smooths motion boundaries of the flow field. Sun et al. [10] proposed a modified weighted median filter (WMF) to prevent this kind of over-smoothing, but the weight is easily influenced by textured, noisy pixels and illumination changes. Recently, Rashwan et al. [22] improved the weight by using the saliency of image gradients to replace the intensity-based measure. Tu et al. [16] proposed a novel combined post-filtering (CPF) method to efficiently remove outliers and handle occlusions simultaneously.

Brox et al. [3] proposed a two nested fixed point iterations based numerical scheme to combine with the coarse-to-fine strategy to efficiently solve the variational methods. As pointed out in [1,2], this method fails to recover many motion details. Because some fine structures are smoothed out at coarse levels and then "forgotten", therefore, they cannot be correctly estimated at final scale. Xu et al. [2] proposed an extended coarse-to-fine (EC2F) refinement framework to reduce the reliance of flow estimates on their initial values propagated from the coarse level. By integrating available matching information into the continuous flow at each scale, the lost motion of fine structures can be recovered.

These methods only consider filtering the intermediate flow fields or refining the initial flow vectors, they neglect to correct the basic elements of the PDEs – the spatiotemporal derivatives.

The correctness of the temporal derivative I_t completely depends on I_{warp} , which means that I_{warp} heavily influences the accuracy of the estimated flow field. Currently, nearly all variational methods (e.g. [10,12,22]) simply use the temporal derivative in the numerical iteration. In contrast, we propose an adaptive guided image filter (AGIF) to filer I_{warp} before computing the temporal derivative. The AGIF technique is especially useful to correct errors of I_{warp} which are caused by outliers and occlusions.

The guided image filter (GIF), which was proposed recently by He et al. [23], smooths the input image by considering the content of a guidance image that can be another image or the input image itself. The GIF is similar to the joint bilateral filter (JBF) [24], which is able to reduce noise while preserve edges. However, the JBF is nonlinear and computationally complex. The computational time of the JBF increases exponentially as the size of the filtering window increases. In contrast, the GIF is implemented as a sequence of box filters, making it linear and efficient. More significantly, its runtime is independent of the filtering window size. Due to these advantages, the GIF is popular in denoising [23,25], sharpness enhancement [26], optical flow computation [28] and stereo matching [27,28]. For example, Xiao et al. [39] applied the GIF to filter the three dimensional cost-volume to preserve edge information and improve operational efficiency. To treat with the problem of flickering-artifacts that is caused by the incoherent disparity maps, Liu et al. [41] proposed a novel temporal consistency enhancement algorithm based on Guided Filter and Temporal Gradient to smooth disparity sequences to improve the consistency of the sequence.

In this work, we present an AGIF technique to smooth I_{warp} to improve the performance of the variational optical flow method. To analyze the characteristic of the GIF as well as the relationship between the reference image and I_{warp} , we reconstruct a guidance image which is a combination of both. The combination is based on a confidence measure which originates from the temporal derivative, as the temporal derivative can be interpreted as an indication of mismatch. Due to this contribution, the filtered I_{warp} can gain useful information from the reference image. Except the general edges, we can correct some intensities which are distorted due to outliers and occlusions of the before filtered I_{warp} . Another contribution is that, we propose a method to select the optimal regularization parameter ε (Eq. (18)) adaptively based on two principles – the error degree of $I_{\mbox{warp}}$ and the size of the input testing images. This method effectively improves the smoothing ability of the GIF.

The paper is organized as follows. Section 2 describes a fundamental CPF method and the numerical optimization process. In Section 3, we propose an AGIF technique to correct errors of the warped interpolation image. Experiments are presented in Section 4. Finally, discussion and conclusions are given in Section 5.

2. Variational optical flow model and minimization

2.1. The combined Post-filtering (CPF) method

Let \mathbf{I}_1 , \mathbf{I}_2 : $(\Omega \subset R^2)$ be the two consecutive frames at time *t* and t+1. $\mathbf{x} = (x, y, t)^T$ denotes the locations of pixels in the spatial image domain Ω , and $\mathbf{w} = (\mathbf{u}, \mathbf{v})^T$ is the flow field describes the displacement in *x*-and *y*-direction between \mathbf{I}_1 and \mathbf{I}_2 . The common assumption of flow estimation is the *brightness constancy assumption* (BCA). Based on the BCA, a data term is formed:

$$E_{\rm D}(\mathbf{w}) = \int_{\Omega} \left| \mathbf{I}_2(\mathbf{x} + \mathbf{w}) - \mathbf{I}_1(\mathbf{x}) \right|^2 d\mathbf{x}$$
(1)

where

$$\mathbf{I}_{warp} = \mathbf{I}_2(\mathbf{X} + \mathbf{W}) \tag{2}$$

is a warped interpolation image with certain interpolation method such as, cubic interpolation, bilinear interpolation [15], or bicubic interpolation [12].

The two unknowns (\mathbf{u}, \mathbf{v}) be determined from one single Eq. (1). To solve this aperture problem, a global smoothness assumption has been introduced, where a smoothness term can be expressed as [8]:

$$E_{\rm S}(\mathbf{w}) = \int_{\Omega} |\nabla \mathbf{u}|^2 + |\nabla \mathbf{v}|^2 d\mathbf{x}$$
(3)

By integrating the smoothness term Eq. (3) into Eq. (1), and by using a parameter λ to steer the relative importance of the two terms, the optical flow estimation can be formulated as an energy minimization problem in which the energy function is:

$$E(\mathbf{w}) = \int_{\Omega} \underbrace{\left| \mathbf{l}_{2}(\mathbf{x} + \mathbf{w}) - \mathbf{l}_{1}(\mathbf{x}) \right|^{2}}_{\text{Data term}} + \lambda \underbrace{\left(\left| \nabla \mathbf{u} \right|^{2} + \left| \nabla \mathbf{v} \right|^{2} \right)}_{\text{Smoothness term}} d\mathbf{x}$$
(4)

This energy function performs poorly when outliers and occlusions occur. A variety of robust penalties [9,10,29] were presented to improve the performance. Recently, Sun et al. [10] pointed out that the L1-norm approximate generalized Charbonnier penalty [30] performs best among them. A generalized Charbonnier penalty based TV-L1 optical flow energy function is defined as:

$$E(\mathbf{w}) = \int_{\Omega} \underbrace{\Psi_{D} \left(\left| \mathbf{I}_{2}(\mathbf{x} + \mathbf{w}) - \mathbf{I}_{1}(\mathbf{x}) \right|^{2} \right)}_{\text{Data term}} + \lambda \underbrace{\Psi_{S} \left(\left| \nabla \mathbf{u} \right|^{2} + \left| \nabla \mathbf{v} \right|^{2} \right)}_{\text{Smoothness term}} d\mathbf{x}$$
(5)

where $\Psi_{\rm D} = \Psi_{\rm S} = \Psi(s^2) = (s^2 + \varsigma^2)^{\alpha}$ ($\alpha = 0.45$), and ς is a small parameter to make sure the energy function differentiable ($\varsigma = 0.001$).

The accuracy can be improved when a MF is applied to smooth the intermediate flow fields during optimization. In contrast to [10], which only applied a WMF for denoising, we adopt a more advanced Classic+CPF method [16] for smoothing. The CPF method is a combined filtering technique, which uses a WMF, a BF and a fast MF to post-smooth the detected edges, occlusions, and the flat regions of the flow field respectively. The CPF based TV-L1 optical flow energy function is expressed as:

$$E(\mathbf{u}, \mathbf{v}, \overline{\mathbf{u}}, \overline{\mathbf{v}}) = \sum_{X, y \in \Omega} \Psi_{D} \left(\left| \mathbf{I}_{2} \left(x + u_{X,y}, y + v_{X,y}, t + 1 \right) - \mathbf{I}_{1} (x, y, t) \right|^{2} \right) \\ + \lambda \cdot \Psi_{S} \left(\left| \nabla u_{X,y} \right|^{2} + \left| \nabla v_{X,y} \right|^{2} \right) + \lambda \cdot 2 \left(\left| \left| \mathbf{u} - \overline{\mathbf{u}} \right| \right|^{2} + \left| \left| \mathbf{v} - \overline{\mathbf{v}} \right| \right|^{2} \right) \\ + \left(\underbrace{\sum_{X_{E}, y_{E}} \sum_{X_{E}', y_{E}' \in N_{X_{E}, y_{E}}} w_{X_{E}, y_{E}, x_{E}', y_{E}'} \left(\left| \overline{u}_{X,y} - \overline{u}_{X', y'} \right| + \left| \overline{v}_{X,y} - \overline{v}_{X', y'} \right| \right) \right|_{X_{E}, y_{E}, x_{E}', y_{E}'} \\ Edge \operatorname{Regions} \rightarrow \operatorname{Weighted} \operatorname{Median Filter} \\ + \underbrace{\sum_{X_{O}, y_{O}} \sum_{XO', yO' \in N_{X_{O}, y_{O}}} w_{X_{O}, yO', yO' (} \left| \overline{u}_{X, y} - \overline{u}_{X', y'} \right| + \left| \overline{v}_{X, y} - \overline{v}_{X', y'} \right| \right) \right|_{X_{O}, y_{O}, x_{O}', y_{O}'} \\ Occlusons \rightarrow \operatorname{Bilateral Filter} \\ + \underbrace{\sum_{X_{F}, y_{F}} \sum_{XF', yF' \in N_{X_{F}, y_{F}}} \operatorname{median} \left(\left| \overline{u}_{X, y} - \overline{u}_{X', y'} \right| + \left| \overline{v}_{X, y} - \overline{v}_{X', y'} \right| \right) \right|_{X_{F}, y_{F}, x_{F}', y_{F}'} \\ \operatorname{Flat} \operatorname{Regions} \rightarrow \operatorname{Median Filter}$$
(6)

where λ and λ_2 are the weight parameters that control the balance of each term. $u_{x,y}$ and $v_{x,y}$ are the elements of **u** and **v** respectively. **ū** and **v** are the auxiliary flow fields of **u** and **v**, and approximate **u** and **v**. $N_{x,y}$ represents the neighborhood of pixel (x, y). The second and the third terms are weighted non-local terms, which conduct a WMF smoothing and a BF smoothing within a specified region of ($\mathbf{\bar{u}}, \mathbf{\bar{v}}$) respectively. $w_{x_E y_E x_E' y_E'}$ is the weight of WMF, which is calculated according to Eq. (9) in [10], and w_{x_0, y_0, x', y'_0} is the weight of BF, which is calculated by using Eq. (3) in [16]. The fourth term conducts a fast median filter (refer to Eq. (2) in [16] for detailed information).

Different from [16], we use the faster Sobel detector to extract the flow field edges, and set standard deviations of the occlusion state function in weights w_{x_E,y_E,x_E,y_E} and $w_{x_0,y_0,x_0',y_0'}$ to σ_d =0.75 and

Table 1

 $\sigma_{\rm e} = 10$. (Table 1 shows the influence of $\sigma_{\rm d}$ and $\sigma_{\rm e}$).

2.2. Minimization

To optimize Eq. (6), we apply an alternating optimization strategy [10] combined with a coarse-to-fine warping scheme [3]. In each pyramid level, at every warping step, two operations are implemented: a well-known fixed point iteration scheme is employed to solve Eq. (5) to get the flow field (\mathbf{u}, \mathbf{v}) , and a CPF method is applied to (\mathbf{u}, \mathbf{v}) to remove outliers.

The fixed point iteration scheme is one of the most successful techniques to solve a large system of PDEs. According to the calculus of variations, a minimizer of Eq. (5) must fulfill the Euler–Lagrange equations:

$$\begin{cases} \Psi_{D}'(\mathbf{I}_{t}^{2})\cdot\mathbf{I}_{x}\mathbf{I}_{t} - \lambda\cdot div \Big(\Psi_{S}'(|\nabla\mathbf{u}|^{2} + |\nabla\mathbf{v}|^{2})\nabla\mathbf{u} \Big) = 0 \\ \Psi_{D}'(\mathbf{I}_{t}^{2})\cdot\mathbf{I}_{y}\mathbf{I}_{t} - \lambda\cdot div \Big(\Psi_{S}'(|\nabla\mathbf{u}|^{2} + |\nabla\mathbf{v}|^{2})\nabla\mathbf{v} \Big) = 0 \end{cases}$$
(7)

where

$$\begin{cases} \mathbf{I}_{x} = \partial_{x}\mathbf{I}_{2}(\mathbf{x} + \mathbf{w}) \\ \mathbf{I}_{y} = \partial_{y}\mathbf{I}_{2}(\mathbf{x} + \mathbf{w}) \\ \mathbf{I}_{t} = \mathbf{I}_{2}(\mathbf{x} + \mathbf{w}) - \mathbf{I}_{1}(\mathbf{x}) \end{cases}$$
(8)

At each pyramid level l ($l \ge 1$), the flow field is updated according to $(\mathbf{u}^{l+1}, \mathbf{v}^{l+1}) = (\mathbf{u}^l + d\mathbf{u}^l, \mathbf{v}^l + d\mathbf{v}^l)$, where $(\mathbf{u}^l, \mathbf{v}^l)$ is a known part from coarse levels and $(d\mathbf{u}^l, d\mathbf{v}^l)$ is an unknown update part that needs to be computed:

$$\begin{cases} \Psi_{D}^{\prime} \left(\mathbf{I}_{t}^{l,k+1}\right)^{2} \cdot \mathbf{I}_{x}^{l,k} \mathbf{I}_{t}^{l,k+1} - \lambda \cdot div \left(\Psi_{S}^{\prime} \left(\left|\nabla \mathbf{u}^{l,k+1}\right|^{2} + \left|\nabla \mathbf{v}^{l,k+1}\right|^{2}\right) \nabla \mathbf{u}^{l,k+1}\right) = 0 \\ \Psi_{D}^{\prime} \left(\mathbf{I}_{t}^{l,k+1}\right)^{2} \cdot \mathbf{I}_{y}^{l,k} \mathbf{I}_{t}^{l,k+1} - \lambda \cdot div \left(\Psi_{S}^{\prime} \left(\left|\nabla \mathbf{u}^{l,k+1}\right|^{2} + \left|\nabla \mathbf{v}^{l,k+1}\right|^{2}\right) \nabla \mathbf{v}^{l,k+1}\right) = 0 \end{cases}$$
(9)

with the abbreviations:

$$\begin{cases} \left(\Psi_{D}^{\prime}\right)^{l,k} = \Psi_{D}^{\prime} \left(\left(\mathbf{I}_{X}^{l,k} d\mathbf{u}^{l,k} + \mathbf{I}_{y}^{l,k} d\mathbf{v}^{l,k} + \mathbf{I}_{t}^{l,k}\right)^{2}\right) \\ \left(\Psi_{S}^{\prime}\right)^{l,k} = \Psi_{S}^{\prime} \left(\left|\nabla\left(\mathbf{u}^{l,k} + d\mathbf{u}^{l,k}\right)\right|^{2} + \left|\nabla\left(\mathbf{v}^{l,k} + d\mathbf{v}^{l,k}\right)\right|^{2}\right) \end{cases}$$
(10)

where *k* is the inner iteration step. To solve the equations, the nonlinear terms of the form $\mathbf{I}_2(\mathbf{x} + \mathbf{w}^{l+1})$ ($\mathbf{w}^{l+1} = (\mathbf{u}^{l+1}, \mathbf{v}^{l+1})$) need to be linearized via Taylor expansion:

$$\mathbf{I}_{t}^{l+1} = \mathbf{I}_{2} (\mathbf{x} + \mathbf{w}^{l+1}) - \mathbf{I}_{1}(\mathbf{x})$$

$$\approx (\mathbf{I}_{2} (\mathbf{x} + \mathbf{w}^{l}) - \mathbf{I}_{1}(\mathbf{x})) + \mathbf{I}_{x}^{l} d\mathbf{u}^{l} + \mathbf{I}_{y}^{l} d\mathbf{v}^{l} = \mathbf{I}_{t}^{l} + \mathbf{I}_{x}^{l} d\mathbf{u}^{l} + \mathbf{I}_{y}^{l} d\mathbf{v}^{l}$$
(11)

The incremental flow $(d\mathbf{u}^{l,k}, d\mathbf{v}^{l,k})$ is solved by the inner fixed point iteration and is refined to $(d\mathbf{u}^{l,k+1}, d\mathbf{v}^{l,k+1})$ iteratively. According to the fixed point iteration scheme, a linear system of PDEs about the unknown incremental flow $(d\mathbf{u}^{l,k+1}, d\mathbf{v}^{l,k+1})$ is formed:

Results (AAE/EPE) of RubberWhale, Venus and Urban3 from the Middlebury dataset with different σ_d and σ_e .

$(\sigma_{\rm d},\sigma_{\rm e})$	(0.30, 20)	(0.30, 10)	(0.5, 10)	(0.75, 15)	(0.75, 10)
RubberWhale	2.240/0.070	2.200/0.070	2.181/0.069	2.205/0.070	2.163/0.069
Venus	3.070/0.223	3.023/0.222	3.025/0.220	3.053/0.223	3.025/0.221
Urban3	2.569/0.370	2.459/0.359	2.460/0.359	2.572/0.366	2.453/0.353
Average	2.626/0.221	2.556/0.218	2.555/0.217	2.610/0.220	2.548/0.215

$$\begin{cases} \left(\Psi_{\hat{D}}\right)^{l,k} \cdot \left(\mathbf{I}_{x}^{l,k}\left(\mathbf{I}_{t}^{l,k}+\mathbf{I}_{x}^{l,k}d\mathbf{u}^{l,k+1}+\mathbf{I}_{y}^{l,k}d\mathbf{v}^{l,k+1}\right)\right) - \lambda \cdot div\left(\left(\Psi_{\hat{S}}\right)^{l,k} \nabla\left(\mathbf{u}^{l,k}+d\mathbf{u}^{l,k+1}\right)\right) = 0 \\ \left(\Psi_{\hat{D}}\right)^{l,k} \cdot \left(\mathbf{I}_{y}^{l,k}\left(\mathbf{I}_{t}^{l,k}+\mathbf{I}_{x}^{l,k}d\mathbf{u}^{l,k+1}+\mathbf{I}_{y}^{l,k}d\mathbf{v}^{l,k+1}\right)\right) - \lambda \cdot div\left(\left(\Psi_{\hat{S}}\right)^{l,k} \nabla\left(\mathbf{v}^{l,k}+d\mathbf{v}^{l,k+1}\right)\right) = 0 \\ (12)$$

We set $d\mathbf{u}^{l,0} = d\mathbf{v}^{l,0} = 0$ for initialization at each pyramid level *l*. $I_x^{l,k}$, $I_y^{l,k}$ and $I_t^{l,k}$ are the intermediate spatio-temporal derivatives. They are usually computed as follows: 1) for the spatial derivatives $I_x^{l,k}$ and $I_y^{l,k}$, we first calculate the *x*-and *y*-direction spatial derivatives of \mathbf{I}_2 with 5-point derivative filter $\frac{1}{12}[-1, 8, 0, -8, 1]$, and then warp the spatial derivatives to \mathbf{I}_1 with the current estimated flow field with respect to the bicubic interpolation [12]; 2) for the temporal derivative $I_t^{l,k}$, it is a difference between the warped interpolation image \mathbf{I}_{warp} and the reference image \mathbf{I}_1 . After computing the spatial and temporal derivatives, the classic successive over-relaxation (*SOR*) method [31] is used to solve Eq. (12) to obtain ($d\mathbf{u}^{l,k+1}$, $d\mathbf{v}^{l,k+1}$).

At the second step, the CPF method is applied to smooth the newly updated flow field $(\mathbf{u}^{l}+d\mathbf{u}^{l,k+1}, \mathbf{v}^{l}+d\mathbf{v}^{l,k+1})$, which can efficiently reduce outliers as well as handle occlusions properly.

One significant problem during the numerical calculation is how to process the spatial and temporal derivatives $I_x^{l,k}$, $I_y^{l,k}$ and $I_t^{l,k}$. Since the intermediate flow field ($\mathbf{u}^{l,k}$, $\mathbf{v}^{l,k}$) is derived directly from the spatiotemporal derivatives, $I_x^{l,k}$, $I_y^{l,k}$ and $I_t^{l,k}$ play a crucial role to determine the accuracy of the final flow field. More importantly, these derivatives all depend on the warping interpolation. Therefore, the final flow field is dependent on the warped interpolation results. Wedel et al. [12] proposed a blended derivative approach to modify the warped spatial derivatives I_x^k and I_y^k . However, to our knowledge, no paper is available to handle the temporal derivative I_t^k . As $\mathbf{I}_t = \mathbf{I}_{warp} - \mathbf{I}_1(\mathbf{x})$, the \mathbf{I}_{warp} must be appropriately tackled. In the next section, we present an AGIF technique to correct errors of \mathbf{I}_{warp} to improve the accuracy of the optical flow estimation.

3. Adaptive guided image filter

Due to the fact that some of the displacement vectors in the intermediate flow field are not accurate, or even completely wrong, their corresponding warped interpolation pixels are not correct. For example, optical flow is undefined at occlusions because of unreliable intensity matching. Therefore, the warped interpolation pixels that come from the undefined flow vectors are wrong. Furthermore, outliers cause incorrect flow vectors and introduce artifacts to I_{warp} . In this section, an AGIF technique is presented to correct these kinds of errors of I_{warp} . Table 3 demonstrates the effectiveness of the proposed AGIF technique.

Recently, GIF was proposed [23] as a high performance image filtering method to effectively reduce noise and preserves edges. Additionally, it performs very fast. Applying the GIF to I_{warp} with a guidance image G, the output F at a pixel i(x, y) is defined as a weighted average:

$$\mathbf{F}_{i} = \frac{1}{\sum_{j \in w_{k}} \mathbf{W}_{i,j}(\mathbf{G})} \sum_{j \in w_{k}} \mathbf{W}_{i,j}(\mathbf{G}) \mathbf{I}_{warp}(j)$$
(13)

where *j* is the index of pixel (*x'*, *y'*), w_k is the kernel window centered at pixel *k*, and the kernel weights function $\mathbf{W}_{i,j}(\mathbf{G})$ can be expressed as:

$$\mathbf{W}_{i,j}(\mathbf{G}) = \frac{1}{|\mathbf{W}|^2} \sum_{k: (i,j) \in w_k} \left(1 + \frac{(\mathbf{G}_i - \mu_k)(\mathbf{G}_j - \mu_k)}{\sigma_k^2 + \epsilon} \right)$$
(14)

where μ_k and σ_k^2 are the mean and variance of the guidance image **G** in a local window w_k , and |w| is the number of pixels in w_k (refer to [23] for more details).

From Eq. (13) we can see that the filtering weights depend on the guidance image **G**. Locally (in a local window w_k), the output **F** obtains approximate details of **G**. Globally, the output **F** is similar to **G**. According to this characteristic, we select the reference image I_1 as the guidance image **G**. The GIF will transfer valid information from I_1 to I_{warp} during the filtering, which is very useful to correct imperfect details of I_{warp} . Some classical techniques to handle occlusions in optical flow estimation in term of label out the occluded pixels and exclude them from the data term [21] or apply diffusion approaches to approximate the occluded flow vectors with their neighbors [32]. In contrast, we correct matching errors due to occlusions or outliers with the application of the GIF – the unreliable intensity matching between I_{warp} and I_1 will be reduced and remedied as the un-occluded useful information in I_1 is propagated to I_{warp} .

However, the **G** only considers the difference between pixel *i* and its neighbors so the error-correction capacity of the traditional GIF is limited. In addition, although we hope I_{warp} should be as similar to I_1 as possible – less interpolation error implies a more accurate estimated flow field [29], I_{warp} must not be equal to I_1 . If I_{warp} equals I_1 , implying that I_t is 0, no valid flow vectors can be computed. Based on this significant feature, we should not simply consider the reference image I_1 but also require to consider I_{warp} for filtering.

Recently, Ham et al. [35] stated that jointly leveraging structural information of guidance image and input image, rather than unilaterally transferring structures of guidance image to the output image, is more effective for denoising. Inspired by the blended technique of [12], which uses the spatial derivatives ∇I_1 and the warped spatial derivatives I_x and I_y to construct a blended version of derivatives $(I_x, I_y) = (1 - \beta) \cdot (I_x, I_y) + \beta \cdot (I_{1x}, I_{1y})$, we blend I_1 and I_{warp} to reconstruct a new guidance image G, which takes into account of the properties of both I_1 and I_{warp} iteratively:

$$\mathbf{G} = \mathbf{W}_{\mathrm{IG}}\mathbf{I}_{\mathrm{warp}} + (1 - \mathbf{W}_{\mathrm{IG}})\mathbf{I}_{\mathrm{I}}$$
(15)

where W_{IG} is a weight to control the relative importance between I_1 and I_{warp} for reconstruction.

The temporal derivative \mathbf{I}_{t} is a good error measure to evaluate the deviation. It is widely used for checking whether the computed flow satisfies the initial model constraints, indicating occlusions [17] and evaluating performances of the optical flow algorithms [29]. In this paper, we use it to compute \mathbf{W}_{IG} :

$$\mathbf{W}_{\rm IG} = \exp\left(-\frac{\mathbf{I}_t^2}{\sigma_{\rm IG}}\right) = \exp\left(-\frac{\left(\mathbf{I}_{\rm warp} - \mathbf{I}_1\right)^2}{\sigma_{\rm IG}}\right)$$
(16)

For one pixel, if the deviation is small, its corresponding estimated flow vector is accurate, thus, large weight is given to \mathbf{I}_{warp} for construction. If the deviation is large, its corresponding estimated flow vector is inaccurate and the interpolation pixel needs to be corrected. Consequently, large weight is given to \mathbf{I}_1 for construction. To keep \mathbf{I}_{warp} as the majority in GIF, we threshold \mathbf{W}_{IG} at each pixel *i*:

$$\mathbf{W}_{\rm IG}(i) = \begin{cases} \mathbf{W}_{\rm IG}(i) & \text{if } \mathbf{W}_{\rm IG}(i) \ge 0.8\\ 0.8 & \text{otherwise} \end{cases}$$
(17)

In general, the W_{IG} should be constrained to [0.6 0.95]. Its actual value is not very significant as we are only interested in keeping I_{warp} as the primary part in the reconstructed guidance image **G**. In this paper, we select 0.8 as the threshold experimentally. In Fig. 1, we use the *Urban3* sequence to illustrate the effect of the proposed GIF method. The threshold weight (Fig. 1(b)) reflects the sequence features better than the un-threshold weight (Fig. 1 (a)). For example, the occlusion status is clearly displayed in Fig. 1 (b). By applying the threshold, at occlusions and edges, the I_{warp} as



Fig. 1. Illustration of the proposed GIF method by using of Urban3 (at the last warping step). (a) The weight W_{IG} without a threshold. (b) The weight W_{IG} with threshold 0.8. (c) The RMS comparison of I_{warp} before and after the application of our GIF. (d) I_{warp} before the application of our GIF method. (e) I_{warp} after the application of our GIF method.

the primary component is integrated into **G** and guides the filtering. In contrast, without the threshold, at some occlusions and edges, **I**₁ is selected as a primary component for constructing the guidance image. However, this deviates the truth since at these regions the structures of **I**_{warp} and **I**₁ are quite different. With the application of our proposed GIF, the occluded error pixels in **I**_{warp} are partly remedied, as shown in Fig. 1(e). It can be also demonstrated in Fig. 1(c) that after using our GIF, the root-mean-square (RMS) error of **I**_{warp} is reduced. Fig. 2 displays the benefits of reconstruction a new guidance image **G** and threshold the weight **W**_{*IG*}.

The regularization parameter ε of the GIF controls the smoothing degree. The larger ε is, the smoother the filtered image will be [23]. According to some classical literatures and various experiments, we find that the optimal parameter ε is related to the

deviation of I_t as well as the size of input images: the larger I_t , the larger ε should be set. The smaller size of the input images, the ε should be also set larger. Based on this observation, an AGIF is proposed to adaptively select the optimal parameter ε .

$$\epsilon = \begin{cases} \min(10^{-4} \cdot 100^{NR} \cdot 10^{ER}, 100) & \text{if } \text{ErrR} < 0.1 \\ \min(10^{-3} \cdot 100^{NR} \cdot 10^{ER}, 100) & \text{else if } \text{ErrR} \ge 0.1 & \text{k} \\ & \text{ErrR} < 0.2 \\ \min(10^{-2} \cdot 100^{NR} \cdot 10^{ER}, 100) & \text{otherwise} \end{cases}$$
(18)

ErrR is an error confidence [33], it represents the degree of error pixels in I_{warp} , and is computed as follows:

$$ErrR = ErrN/(H.W)$$
(19)



Fig. 2. Results (at the last warping step) of Urban3. (a) The filtered I_{warp} in term of guidance image I_1 . (b) The filtered I_{warp} in term of guidance image I_{warp} . (c) The filtered I_{warp} in term of the reconstructed guidance image **G** without threshold **W**_{IG}. (d) The filtered I_{warp} in term of **G** with threshold **W**_{IG}. (e) The AAE and EPE of the 4 different GIF strategies.

ErrN is the number of error pixels in I_{warp} , and H and W are the height and width of I_{warp} respectively. Bruhn and Weickert [33] pointed out that at locations where the deviation is small, the computed flow is accurate. At locations where the deviation is large, the optical flow model assumptions are violated severely. According to this statement, we consider one pixel to be erroneous if $W_{IG}(i) < 0.8$.

ER is an error confidence [29,36], which represents the RMS of $I_{t.}$ and it is given by:

$$ER = round(RMS(I_t)/10)$$
(20)

where round is the mathematical round operation. The RMS of I_t is the basic and widely measure to evaluate the performance of variational optical flow algorithms without knowing the ground truth [29,36]. In theory, the worse the flow field is, the larger the RMS(I_t) becomes, and the better the flow field is, the smaller the RMS(I_t) gets. This is because the more accurate of the estimated flow field (**u**, **v**), the better match that can be achieved between the two frames, and the smaller of the RMS(I_t) will be.

NR is a measure of the size of the input images. If the ratios of noise in two different sized images are the same, from visual, the small sized image appears more noisy. From the mathematical side, their probability to capture noise is different. It is much easier to get noise in the small size image. In other words, the small size image looks like noisier. Hence, for the smaller image, the parameter ε should be set larger. In this paper, we select the resolution 640 480 as a fixed divide for all the experiments [37]. NR is computed:

$$NR = \max(0, \operatorname{round}(640.480/H/W) - 1)$$
(21)

This strategy can adaptively select a proper parameter ε to improve the smoothing. As shown in Fig. 3, noise at smooth areas are reduced with the adaptive ε selection strategy, and the motion boundary over-smoothing problem is also reduced. Furthermore, in Table 2, we test the influence of the proposed size fact NR quantitatively. By comparing the results of AGIF (NoSize) i.e. deleting the NR from ε , with AGIF of the 4 small size sequences, we can find that with the contribution of the proposed size fact NR, the estimation accuracy is increased (Table 3).

4. Implementation and experimental results

4.1. Implementation

To be robust against illumination changes, we preprocess the input images by applying the Rudin–Osher–Fatemi (ROF) [26] based structure-texture decomposition [12]. The incremental coarse-to-fine technique in combination with the fixed pointed iteration scheme is used to estimate the optical flow. In the outer

Table 2

AAE/EPE results of four training sequences from the Middlebury dataset and the UCL dataset v1.2.

	Venus	YoesmiteSun	Mayan1	Mayan2
GIF	3.206/0.230	2.691/0.150	2.460/0.909	1.391/0.238
AGIF(NoSize)	3.161/0.228	2.705/0.151	2.447/0.911	1.403/0.233
AGIF	3.028/0.221	2.484/0.132	2.398/0.895	1.400/0.229

loop of the optimization, a graduated nonconvexity (GNC) strategy is used [34], which linearly combines a quadratic penalty function with a generalized Charbonnier penalty function [10]: $E_{\gamma} = (1-\gamma)$ $E_Q(u, v) + \gamma E_C(u, v)$. According to [10,38], we implement 3 GNC steps with $\gamma = \{0, 0.5, 1\}$. In building the pyramid, we use a downsampling factor of 2/3 (see Table 4), and the number of pyramid levels is adaptively determined with a constraint that the coarsest level has a height or width around 20–30 pixels. Referring the source code of the WLIF-Flow method [38] for more detail, which is available at: www.projects.science.uu.nl/opticalflow/ WLIF-Flow/.

At each warping step, we apply the CPF method [16] to denoise the intermediate flow field: a 7×7 WMF is used to smooth the Sobel detected edge regions, a 9×9 BF is used to smooth the occluded regions and a 5×5 fast MF is used to smooth other flat regions. Look Fig. 4 to overview the proposed method. All the experiments have been executed on a PC with an Intel Core i5-2410M 2.30 GHz processor and 4 GB memory, and the pixel values of the experimental images are in the range [0, 255].

4.2. Experimental results

This paper focuses on using an AGIF technique to smooth I_{warp} to correct errors that may be caused by outliers and occlusions during numerical iterations. For a quantitative measurement, we first test our method on eight training sequences from the Middlebury dataset [29]. Table 5 shows the average angular error (AAE) and the average end-point error (EPE) of four methods: the proposed method, referred to as AGIF+OF, the relevant method GIF+OF (i.e. without adaptive ε selection strategy), the Classic+NL method [10] and the Classic+CPF method [16]. The four methods have similar frameworks - the same data term and the smoothness term, and the similar non-local smoothing strategies. Comparing our AGIF+OF method to both the Classic+NL method and the Classic+CPF, we can find that with the application of the AGIF technique, the accuracy of the estimated flow field is improved. Especially for some complex sequences, such as RubberWhale, Venus and Urban3 (contain an amount of outliers and occlusions), the AAE and EPE of our method is much lower than other two methods. Additionally, Table 5 compares the performance of the GIF technique with the AGIF technique. Clearly, our AGIF



Fig. 3. Comparison of the estimated flow fields on the Venus sequence (from Middlebury dataset) and the YoesmiteSun sequence (from UCL dataset v1.2). (a, b) Estimated flow fields of Venus with the GIF and the AGIF respectively. (c, d) Estimated flow fields of YoesmiteSun with the GIF and the AGIF respectively.

 Table 3

 AAE/EPE results of eight training sequences from the Middlebury dataset without the CPF and GNC operations.

	Urban2	Urban3	Grove2	Grove3	RubberWhale	Venus	Dimetrodon	Hydrangea
None	2.208/0.278	5.311/0.609	1.825/0.133	7.974/0.842	3.057/0.097	5.459/0.331	2.589/0.133	1.888/0.172
GIF	2.210/0.279	5.260/0.603	1.806/0.126	7.956/ 0.832	3.027/0.096	5.457/0.328	2.579/ 0.131	1.883/0.171
AGIF (With)	2.190/0.270	5.252/0.600	1.801/0.121	7.901/0.833	3.012/0.096	5.417/0.328	2.563/0.132	1.892/ 0.169

outperforms GIF, which demonstrates our strategy to adaptively select a proper ε is beneficial for improving smoothing. Table 5 shows the RMS errors of I_{warp} before AGIF and after AGIF at the final warping step of the last iteration. After using the AGIF, some errors in I_{warp} were corrected. Besides, in the whole coarse-to-fine and iteration procedure, the GIF will be implemented many times, and the accumulated difference is large. Therefore, the preciseness of the temporal derivative \mathbf{I}_{t} is improved and the final optical flow accuracy can be largely improved. It can be further demonstrated in Table 6, where we compare the GIF with our AGIF in terms of the training sequences from the MPI-Sintel dataset (using frame 10 and frame 11 for testing). The AAE/EPE results of our AGIF are more accurate than the results of GIF. Fig. 5 shows the AGIF technique has the correction ability from a visual aspect. By comparing the corresponding flow fields of each sequence, especially at occlusions and discontinuities (red rectangle highlighted regions), we find that errors are reduced with the application of our AGIF technique. Like the Urban3, the occlusion caused errors, are clearly seen in Fig. 5(f)-(h), while they are effectively remedied in Fig. 5(i).

Furthermore, we test our method on some sequences from the MIT dataset (available at http://people.csail.mit.edu/celiu/motio nAnnotation/) and the UCL dataset (available at http://visual.cs.ucl. ac.uk/ pubs/flowConfidence/supp/indx.html). Table 7 shows the AAE and EPE of three methods: our AGIF+OF method, the Classic+NL [10] and the Classic+CPF [16]. Both the lowest AAE and EPE of our method verify the validity of the proposed AGIF technique. For some sequences, such as *Robot*, the improvement even reaches to 50%. Fig. 6 shows the visual flow fields of sequences fish, Cameramotion, Crates2 and drop1Txtr1 with respect to these three methods. For the fish sequence, we can see a large number of noise and outliers that are distributed in the fields of the Classic+NL and the Classic+CPF algorithms, in contrast, the noise and outliers are greatly reduced in our flow field. Besides, the motion parts of the four fishes are clearly estimated. For the Cameramotion sequence, the errors caused by occlusion around the rear-vision mirror region are corrected. For others: Crates2 and drop1Txtr1, our AGIF technique also has high-performance. One remarkable discovery is that for all the four tested sequences, outliers are smoothed out, occluded parts are adequately recovered, and boundaries are well preserved.

We evaluate our AGIF+OF method on the Middlebury benchmark. Fig. 7 shows that both the AAE and the EPE rank 15th at the time of submission, and the average normalized interpolation error (ANIE) is more accurate than some methods which only have a good performance on the AAE and EPE. Most importantly, we can see that our method outperforms nearly all the TV-L1 non-local (NL) approaches, such as the Efficient-NL algorithm, the Classic+NL algorithm, the NL-TV-NCC algorithm and the OcclusionTV-L1 algorithm (refer to the Middlebury testing website for more details). Moreover, our AGIF technique is beneficial for all Euler–Lagrange based variational algorithms which apply both the GNC method and the warping strategy. Table 8 shows the performance of the classical HS [8] method and the BA [9] method is significantly improved when our AGIF is used.

We also evaluate our AGIF+OF method on the new MPI-Sintel [14] dataset. From Table 9, we can see that our method is much better than the Classic+NL [10] method. More importantly, on the clean testing part, our AGIF+OF method outperforms the MDP-Flow2 method [2], which is the topmost algorithm on the Middlebury testing. The MDP-Flow2 method has similar idea as us, which focuses on reducing initial flow errors and preserving motion details during the coarse-to-fine refinement framework. The result demonstrates that correcting errors of I_{warp} during optimization with our AGIF technique is valid and useful.

Fig. 8 shows three examples from the Middlebury real sequences Backvard, Basketball, and MiniCooper. We can find that the proposed AGIF+OF method is effective to handle some complex real conditions (comparing the red rectangle labeled regions of the flow fields of three methods). For the Backyard, because of the occlusion, the motion of the right foot of the little boy, the right leg and left foot of the oldest girl, and the right leg of the little girl who is hold, is lost or is wrong in Fig. 8(a) and (b). In contrast, the motion of these parts is recovered in Fig. 8(c). For the Basketball, as shown in Fig. 8(f), the motion of the arms and hands of the right person is correctly estimated, and his profile is clearly recovered that we can easily discern his hair, noise and mouth. The motion of the basketball and its corresponding shadow are also correctly computed. However, the estimated flow vectors of these parts contain some serious errors with the other two methods [10,16]. For example, the motion of the shadow of the basketball is confused with the background in Fig. 8(d), while some detailed motion information of the profile of the right person is oversmoothed in Fig. 8(e). For the MiniCooper, it is hard to capture the motion of the head and arms of the man due to noise and occlusion. Therefore, some important motion of these parts are unrecovered in Fig. 8(g) and (h). Fig. 8(i) shows that these problems are properly tackled with the AGIF technique: the motion boundary is sharp enough to distinguish the man and the outline of the window of the trunk is clearly reflected. To be specific, the motion of the forehead and the right arm is lost in Fig. 8(g) and (h), while is correctly estimated in Fig. 8(i).

Fig. 9 shows the results of the challenging *Football* sequence. For the *Football* sequence, errors are easily generated at regions of the player's right foot and hand, as well as the football. Comparing Fig. 9(c) with (d), we can find that although the state-of-the-art MDP-Flow2 method [2] can recover the motion at these regions, the motion information, especially the motion boundaries, is very

Table 4

lubic 1								
AE/EPE resul	lts of eight	training seque	nces from t	the Middlebury	dataset with	different	downsampling fac	tors.

	Urban2 Urban3 Grove2 1.889/0.213 2.512/0.387 1.306/0.092 1.894/0.211 3.211/0.457 1.307/0.092		Grove3	RubberWhale	Venus	Dimetrodon	Hydrangea	
Factor=1/2	1.889/0.213	2.512/0.387	1.306/0.092	4.953/0.481	2.231/0.071	3.073/0.223	2.468/0.127	1.887/0.159
Factor=4/5	1.894/0.211	3.211/0.457	1.307/0.092	4.454/0.449	2.226/0.070	3.002/0.220	2.472/0.127	1.887/0.158
Factor=2/3	1.825/0.204	2.465/0.357	1.272/0.090	4.373/0.437	2.166/0.069	3.028/0.221	2.429/0.125	1.861/0.155

```
Input: two images I_1 and I_2
Preprocess I1 and I2 (structure-texture decomposition)
for l = 1 to max level (automatically compute)
  Compute pyramid images {}^{I}I_{I} and {}^{I}I_{2};
end
Initialize
W = 0, and L = max level; (W = (u, v) is the flow field)
for iGNC = 1 to gnc_iteration
  for l = L to 1
       for k = 1 to warp iteration;
            if l == 1
               warp iteration = warp iteration +2
            end
            Initialize d\mathbf{W}^{l,k} = 0; (d\mathbf{W}^{l,k} = (d\mathbf{u}, d\mathbf{v})^{l,k} is the incremental flow field, see Eq. (11))
            Set \mathbf{W}^{l} = \mathbf{W}^{l} + d\mathbf{W}^{l,k} (before compute d\mathbf{W}^{l,k})
            if l==1 && k>1
               Perform AGIF
            else
               Perform GIF
            end
            Compute d\mathbf{W}^{l,k} by solving Eq. (12) with SOR solver
            Update \mathbf{W}^l = \mathbf{W}^l + d\mathbf{W}^{l,k}
            Perform CPF to \mathbf{W}^{l}
        end
    end
end
Output: optical flow field W (u, v)
```

Fig. 4. Method overview.

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Table	J

AAE/EPE results of eight training sequences from the Middlebury dataset and the RMS error of I_{warp} of the proposed AGIF+OF method.

	Urban2	Urban3	Grove2	Grove3	RubberWhale	Venus	Dimetrodon	Hydrangea
Classic+NL [3] Classic+CPF [13] GIF+OF AGIF+OF	2.011/0.211 1.866/ 0.203 1.896/0.219 1.825 /0.204	2.862/0.413 2.604/0.402 2.438 /0.384 2.465/ 0.357	1.413/0.098 1.323/0.092 1.319/0.092 1.272/0.090	4.833/0.458 4.686/0.456 4.920/0.476 4.373/0.437	2.327/0.072 2.285/0.071 2.231/0.071 2.166/0.069	3.256/0.232 3.181/0.229 3.206/0.230 3.028/0.221	2.476/0.127 2.491/0.127 2.468/0.127 2.429/0.125	1.816/0.151 1.856/0.154 1.880/0.157 1.861/0.155
I _{warp} (Before AGIF) I _{warp} (After AGIF)	RMS 17.2176 16.6114	RMS 24.9190 24.2727	RMS 17.1546 15.8344	RMS 25.4570 23.8594	RMS 9.6728 9.1588	RMS 17.2574 16.2729	RMS 8.9276 8.5278	RMS 15.4837 14.5161

AAE/EPE results of the training sequences (both clean pass and final pass) from the MPI-Sintel dataset.

Clean	alley_2	ambush_2	bamboo_2	cave_2	market_2	shaman_2	sleeping_2	temple_2
GIF+OF AGIF+OF Final	2.266/0.208 2.131/0.205	13.409/7.088 13.379/7.075	5.175/0.518 5.138/0.501	3.029/0.922 3.020/0.913	6.056/1.048 5.906/1.035	4.363/0.317 4.258/0.301	1.295/0.060 1.273/0.052	4.000/0.707 3.904/0.675
GIF+OF AGIF+OF	2.242/0.171 2.223/0.168	47.840/28.817 47.705/28.798	5.503/0.598 5.493/0.584	3.321/1.188 3.211/1.152	7.864/1.359 7.797/1.332	4.965/0.337 4.895/0.318	1.336/0.062 1.321/0.051	7.552/1.329 7.413/1.293

dim (see Fig. 9(d)). For example, the motion boundaries of the right fingers of the player is smoothened too much to discern, and also the motion boundary of the football is not distinctive. In contrast, our AGIF+OF method (see Fig. 9(c)) corrects these defects. The motion boundaries of the fingers are well preserved. The outline of the football is very clear. Additionally, we can even distinguish the right shoe of the player. All in all, from the well preserved motion boundaries of our flow field, we can see that our AGIF technique is good at correcting motion errors.

(a) and (c), our AGIF+OF method correctly estimates the motion fields of them. In Fig. 10(a), the motion of the car, and the motion of the trees at the right side are distinctive. In Fig. 10(c), the motions of the two cars, the rider and even his shadow, and the backgrounds (e.g. houses and the mountain-like building) are all clearly recovered. The warped images, as shown in Fig. 10(b) and (d), are also correct.

In the last experiment, we select the KITTI complex outdoor sequences dataset for testing (Available at http://www.cvlibs.net/data sets/kitti/eval_stereo_flow.php?benchmark=flow). As shown in Fig. 10

5. Discussion and conclusions

This paper proposed an effective AGIF technique to improve the performance of the variational optical flow method. It corrects



Fig. 5. Visual comparison of the estimated flow fields on sequences Venus and Urban3 (each row, from left to right). (a, f) Estimated flow fields of the Classic+NL[10]; (b, g) Estimated flow fields of the Classic+CPF[16]; (c, h) Estimated flow fields without using the proposed AGIF technique. (d, i) Estimated flow fields with using the proposed AGIF technique. (e, j) Corresponding ground-truths.

Table 7

AAE/EPE results of training sequences from the MIT dataset and the UCL dataset.

	Fish	Cameramotion	Table	Crates2	Robot	Brickbox1t1	blow1Txtr1	drop1Txtr1
Classic+NL	20.574/0.837	6.819/0.592	4.032/1.197	16.614/10.301	9.276/1.534	0.657/0.225	0.528/0.026	1.329/0.050
Classic+CPF	15.306/0.627	5.785/0.533	3.750/1.140	16.458/10.390	8.898/1.514	0.618/0.237	0.428/0.023	1.010/0.041
AGIF+OF	12.325/0.505	5.220/0.499	3.618/1.237	13.156/2.421	4.064/0.886	0.525/0.190	0.407/0.020	0.898/0.036
Improvement	19.48%/19.46%	9.77%/6.38%	3.52%/8.51%	20.06%/76.7%%	54.33%/41.48%%	15.05%/30.4%	4.91%/13.04%	11.09%/12.2%



Fig. 6. Visual comparison of the estimated flow fields on sequences fish, Cameramotion, Crates2 and drop1Txtr1 (each row, from left to right). (a), (d), (g) and (j) Estimated flow fields of the method Classic+NL[10]. (b), (e), (h) and (k) Estimated flow fields of the method Classic+CPF[16]. (c), (f), (i) and (l) Estimated flow fields of the proposed method AGIF+OF.

а

Average		Army	Mequon	Schefflera	Wooden	Grove	Urban	Yosemite	Teddy
angle		(Hidden texture)	(Hidden texture)	(Hidden texture)	(Hidden texture)	(Synthetic)	(Synthetic)	(Synthetic)	(Stereo)
error	avg.	GT im0 im1	<u>GT im0 im1</u>	<u>GT im0 im1</u>	<u>GT im0 im1</u>	GT im0 im1	<u>GT im0 im1</u>	<u>GT im0 im1</u>	<u>GT im0 im1</u>
	rank	all disc untext							
AGIF+OF [92]	19.8	3.06 14 8.20 13 2.55 16	3.17 40 10.6 33 2.46 39	3.46 21 8.97 21 2.24 26	2.61 10 13.7 14 1.33 13	2.63 12 3.46 10 2.11 16	2.88 11 8.34 10 2.35 8	2.10 16 3.56 27 2.09 28	1.80 25 3.68 24 2.24 29
Efficient-NL [60]	20.0	2.99 11 8.23 16 2.28 7	2.72 21 8.95 21 2.25 25	3.81 29 9.87 28 2.07 18	2.77 22 14.3 21 1.46 24	2.61 10 3.48 11 1.96 9	3.31 21 8.33 9 2.59 15	2.60 40 3.75 35 2.54 41	1.60 18 3.02 11 1.66 17
ALD-Flow [67]	20.3	2.82 5 7.86 7 2.16 5	2.84 27 10.1 27 1.86 15	3.73 27 10.4 30 1.67 10	3.10 31 16.8 40 1.28 9	2.69 15 3.60 18 1.85 7	2.79 8 11.3 28 2.32 7	2.07 13 3.25 7 3.10 62	2.03 32 5.11 34 1.94 24
IROF++ [58]	21.4	3.17 22 8.69 23 2.61 19	2.79 24 9.61 24 2.33 28	3.43 18 8.86 18 2.38 31	2.87 25 14.8 25 1.52 30	2.74 17 3.57 17 2.19 19	3.20 18 9.70 20 2.71 20	1.96 9 3.45 16 1.22 5	1.80 25 4.06 26 2.50 35
SCR [73]	21.7	3.12 16 8.48 19 2.59 17	2.95 33 10.4 30 2.35 30	3.19 12 8.09 12 2.43 35	2.63 13 13.9 17 1.35 15	2.81 20 3.64 20 2.30 22	3.02 13 8.29 8 2.39 12	2.77 48 3.79 38 2.89 54	1.39 12 2.85 9 1.60 16
Classic+CPF [90]	22.3	3.14 18 8.60 21 2.63 23	3.03 38 10.6 33 2.33 28	3.66 24 9.58 24 2.20 25	2.61 10 14.1 19 1.34 14	2.68 14 3.53 14 2.21 20	2.85 9 7.95 7 2.38 9	2.44 34 3.49 21 2.90 56	1.67 21 3.40 20 2.43 33
TC-Flow [46]	23.0	2.91 7 8.00 10 2.34 11	2.18 7 8.77 18 1.52 1	3.84 31 10.7 34 1.49 1	3.13 32 16.6 38 1.46 24	2.78 19 3.73 25 1.96 9	3.08 15 11.4 29 2.66 16	1.94 7 3.43 15 3.20 67	3.06 39 7.04 38 4.08 60
Sparse-NonSparse [56]	23.0	3.14 18 8.75 25 2.76 29	3.02 36 10.6 33 2.43 34	3.45 20 8.96 19 2.36 29	2.66 15 13.7 14 1.42 18	2.85 24 3.75 26 2.33 24	3.28 20 9.40 17 2.73 21	2.42 31 3.31 9 2.69 44	1.47 14 3.07 16 1.66 17
LSM [39]	24.6	3.12 16 8.62 22 2.75 28	3.00 35 10.5 32 2.44 36	3.43 18 8.85 17 2.35 28	2.66 15 13.6 12 1.44 20	2.82 21 3.68 21 2.36 26	3.38 24 9.41 18 2.81 25	2.69 44 3.52 24 2.84 49	1.59 17 3.38 19 1.80 23
Correlation Flow [78]	24.7	3.38 32 8.40 18 2.64 24	2.23 87.54 10 1.56 3	5.14 42 13.1 41 1.60 6	2.09 4 8.15 1 1.35 15	3.12 33 4.09 36 2.34 25	4.01 42 11.5 31 4.00 52	2.59 39 3.61 30 3.00 61	1.49 15 3.04 13 1.42 12
Ramp [62]	25.5	3.18 24 8.83 26 2.73 27	2.89 30 10.1 27 2.44 36	3.27 15 8.43 15 2.38 31	2.74 21 14.2 20 1.46 24	2.82 21 3.69 23 2.29 21	3.37 23 9.31 16 2.93 29	2.62 42 3.38 14 3.19 66	1.54 16 3.21 17 2.24 29
PMF [75]	26.0	3.61 36 9.07 27 2.62 21	2.40 11 8.05 11 1.83 14	2.61 8 6.27 8 1.65 9	3.35 40 15.4 29 1.58 33	2.54 7 3.27 7 1.71 5	3.59 34 11.1 27 3.46 43	4.07 77 6.18 85 4.02 77	1.06 3 2.38 3 1.25 9
COFM [59]	26.3	3.17 22 9.90 41 2.46 13	2.41 13 8.34 15 1.92 16	3.77 28 10.5 31 2.54 40	2.71 20 14.9 27 1.19 7	3.08 32 3.92 30 3.25 58	3.83 37 10.9 24 3.15 35	2.20 23 3.35 10 2.91 58	1.62 20 2.56 5 2.09 26
Classic+NL [31]	28.0	3.20 25 8.72 24 2.81 31	3.02 36 10.6 33 2.44 36	3.46 21 8.84 16 2.38 31	2.78 23 14.3 21 1.46 24	2.83 23 3.68 21 2.31 23	3.40 26 9.09 15 2.76 23	2.87 52 3.82 41 2.86 52	1.67 21 3.53 22 2.26 32
TV-L1-MCT [64]	28.6	3.16 21 8.48 19 2.71 26	3.28 48 10.8 42 2.60 48	3.95 33 10.5 31 2.38 31	2.69 18 13.9 17 1.45 23	2.94 28 3.79 27 2.63 42	3.50 31 9.75 21 3.06 33	2.08 14 3.35 10 2.29 32	<u>1.95</u> 29 3.89 25 2.71 38

b

Average endpoint		(Hid	Arm	/ xture)	(H	Meq	uon	S (Hi	cheff	lera	(Hic	Noode	en sture)	(5	Grove	e ic)	(Urbar Synthet	1 ic)	Yosemite (Synthetic)			Teddy (Stereo)		
error	avg.	GT im0 im1			GT im0 im1			G	<u>im0</u>	im1	GT	im0	im1	GT	GT im0 im1		GÌ	im0	im1	GT im0 in		<u>im1</u>	GT i		<u>im1</u>
	rank	all	<u>disc</u>	untex	t <u>all</u>	dis	<u>sc</u> untex	t <u>all</u>	disc	untext	all	<u>disc</u>	untext	all	<u>disc</u>	<u>untext</u>	all	<u>disc</u>	untext	all	<u>disc</u>	<u>untext</u>	all	<u>disc</u>	<u>untext</u>
AGIF+OF [92]	20.2	0.08	0.22	3 0.07 1	0.23	10.7	3 <mark>33 0.18 3</mark>	3 <u>0.28</u> 1	0.66	22 0.18 1	0.147	0.70	9 0.08 11	0.57 7	0.85	7 0.38 10	<u>0.47</u> 2	0.97 1	1 0.31 21	0.13 30	0.132	6 0.22 33	0.51 29	0.99 2	3 0.74 34
TC-Flow [46]	21.7	0.07	0.21	3 0.06	<u>0.15</u>	10.5	9 ₁₀ 0.11	1 <u>0.31</u> 3	0.78	34 0.14	0.16 28	0.86 3	1 0.08 11	0.75 31	1.11 3	3 0.54 <mark>2</mark> 9	0.42 1	1.40 4	3 0.25 7	0.11 11	0.12	8 O.29 61	0.62 37	1.35 3	7 0.93 49
FESL [74]	21.7	0.08	0.21	30.071	0.25	9 0.7	5 <mark>39 0.19 4</mark>	2 0.27 1	0.61	14 0.18 1	0.147	0.68	7 0.08 11	0.61 12	0.89 1	0.44 15	0.47 2	1.03 1	6 0.32 <mark>2</mark> 5	0.14 39	0.15 5	4 0.25 45	0.50 24	0.96 1	7 0.63 12
Classic+CPF [90]	21.7	0.08	0.23	8 0.07 1	0 <u>.22</u>	29 0.7	3 <mark>33 0.17 2</mark>	4 <u>0.30</u> 2	0.70	25 0.18 1	0.14 7	0.72 1	6 0.08 11	0.63 13	0.93 14	4 0.45 17	<u>0.51</u> 3	1.03 1	6 0.32 <mark>2</mark> 5	0 <u>.14</u> 39	0.12	8 0.30 <mark>66</mark>	<u>0.48</u> 18	0.93 1	3 0.72 <mark>26</mark>
ALD-Flow [67]	21.8	0.07	0.21	3 0.06	5 <u>0.19</u> ·	13 O.6	4 <mark>20</mark> 0.13 1	0 <u>.30</u> 2	0.73	0.15	0.17 33	0.92 4	3 0.07 <mark>4</mark>	<u>0.78</u> 34	1.14 34	4 0.59 34	0.33	1.30 3	5 0.21 3	0.12 20	0.12	8 O.28 56	0.54 33	1.193	5 0.73 <mark>3</mark> 0
SCR [73]	21.8	0.087	0.23	8 0.07 1	0 <u>.22</u>	<mark>9 0.7</mark>	1 29 0.17 2	4 0.27 1	0.60	13 0.19 2	0.147	0.731	7 0.08 11	0.63 13	0.92 1	3 0.44 15	<u>0.51</u> 3	1.08 2	1 0.33 30	0.15 49	0.132	6 O.29 61	0.47 15	0.931	3 0.67 20
COFM [59]	22.1	0.08	0.26	5 0.06	<u>0.18</u>	90.6	2 15 0.14 1	4 <u>0.30</u> 2	0.74	31 0.19 2	0.15 17	0.86 3	1 0.07 4	<u>0.79</u> 35	1.14 34	4 0.74 50	0.35 10	0.87	7 0.28 11	<u>0.14</u> 39	0.12	8 0.28 5 6	<u>0.49</u> 20	0.94 1	5 0.71 <u>25</u>
Sparse-NonSparse [56]	22.3	0.08	0.23	8 0.07 1	0.22	29 0.7	3 <mark>33 0.18 3</mark>	3 <u>0.28</u> 1	0.64	17 0.19 2	0.147	0.71 1	3 0.08 11	0.67 22	0.99 2	3 0.48 21	<u>0.49</u> 3	1.06 1	8 0.32 25	0.14 39	0.11	10.28 56	0.49 20	0.98 2	0.73 30
Efficient-NL [60]	22.5	0.087	0.22	3 0.06	<u>0.21</u> :	24 0.6	7 22 0.17 2	4 <u>0.31</u> 3	0.73	29 0.18 1	0.147	0.71 1	3 0.08 11	<u>0.59</u> 9	0.88	9 0.39 11	1.30 6	1.35 3	8 0.67 <mark>6</mark> 8	0.14 39	0.132	6 0.26 47	0.45 11	0.85	6 0.55 7
LSM [39]	23.8	0.087	0.23	8 0.07 1	0.22	<mark>9 0.7</mark>	3 <mark>33 0.18 3</mark>	3 <u>0.28</u> 1	0.64	17 0.19 2	0.147	0.70	9 0.09 23	0.66 18	0.97 1	9 0.48 21	0.50 3	1.061	8 0.33 <mark>3</mark> 0	0.15 49	0.12	8 0.29 61	0.50 24	0.99 2	3 0.73 30
Ramp [62]	24.4	0.087	0.24	6 0.07 1	0.21	24 0.7	2 <mark>31 0.18 3</mark>	3 <u>0.27</u> 1	0.62	15 0.19 2	0.15 17	0.71 1	3 0.09 <mark>2</mark> 3	0.66 18	0.97 1	9 0.49 23	<u>0.51</u> 3	1.09 2	3 0.34 <mark>3</mark> 6	0.15 49	0.12	8 0.30 66	0.48 18	0.96 1	7 0.72 26
Classic+NL [31]	26.3	0.087	0.23	80.071	0.22	29 O.74	4 <mark>37 0.18 3</mark>	3 <u>0.29</u> 2	20.65	21 0.19 2	0.15 17	0.73 1	7 0.09 23	0.64 17	0.93 14	4 0.47 18	<u>0.52</u> 43	1.12 2	<mark>5 0.33 30</mark>	0.16 59	0.132	6 0.29 <mark>61</mark>	0.49 20	0.98 2	0 0.74 34
TV-L1-MCT [64]	26.5	0.087	0.23	8 0.07 1	<u>0.24</u>	15 0.7	7 41 0.19 4	2 <u>0.32</u> 3	0.76	33 0.19 2	0.147	0.69	8 0.09 <mark>2</mark> 3	<u>0.72</u> 28	1.03 24	4 0.60 35	0.54 44	1.10 2	4 0.35 37	0.11 11	0.12	8 0.20 22	0.54 33	1.04 3	0 0.84 41

С

Average		1	Veque	on	S	cheffl	era		Urbar	ı		Tedd	у	Ba	ackya	rd	Ba	sketb	all	Du	mptru	ick	Ev	ergre	en
normalized interpolation		(Hid	den tex	xture)	(Hid	dden te	xture)	(\$	Synthet	ic)		(Stered)	(High-s	peed c	camera)	(High-s	peed c	amera)	(High-s	peed c	amera)	(High-s	peed c	amera)
error	avg.	im	<u> GT</u>	im1	im	<u>) GT</u>	im1	im0	<u>GT</u>	im1	im	<u>) GT</u>	im1	im0	<u>GT</u>	im1	<u>im0</u>	GT	im1	im0	<u>GT</u>	im1	<u>im0</u>	<u>GT</u>	im1
	rank	all	<u>disc</u>	untext	all	<u>disc</u>	untext	all	<u>disc</u>	untext	t <u>all</u>	<u>disc</u>	untext	all	<u>disc</u>	<u>untext</u>	all	<u>disc</u>	untext	all	<u>disc</u>	untext	all	<u>disc</u>	untext
AGIF+OF [92]	40.0	<u>0.61</u> 16	0.78 1	7 0.64 3	<u>0.67</u> 21	0.92	21 0.61 19	<u>1.17</u> 40	2.07 5	7 1.37 4	4 <u>0.97</u> 20	1.06 2	1 1.25 35	1.20 71	1.30 71	1.69 72	<u>1.18</u> 50	1.75 <mark>57</mark>	1.07 24	0.75 28	1.44 32	0.72 29	<u>1.13</u> 89	1.72 90	0.64 34
Classic++ [32]	40.1	<u>0.62</u> 32	0.80 2	6 0.66 47	0.78 47	1.10	7 0.66 45	0.93 14	1.36 1	0.75	6 <u>1.04</u> 51	1.124	<mark>5 1.28 55</mark>	1.08 34 1	1.11 34	4 1.18 32	<u>1.18</u> 50	1.6 <mark>9 5</mark> 0	1.10 44	<u>0.89</u> 55	1.86 <mark>58</mark>	0.72 29	<u>0.99</u> 55	1.47 se	0.64 34
Sparse Occlusion [54]	40.8	<u>0.63</u> 47	0.87 5	4 0.65 26	<u>0.77</u> 44	1.11	50 O.63 34	<u>0.91</u> 9	1.45 1	90.86 1	5 <u>0.96</u> 16	1.08 2	6 1.21 8	1.21 78	1.32 77	7 1.69 72	<u>1.14</u> 36	1.63 44	1.12 56	0.86 50	1.77 53	0.71 24	<u>1.04</u> 69	1.56 😆	0.634
Ramp [62]	41.2	0.62 32	0.82 3	6 0.65 <u>26</u>	0.66 16	0.901	16 0.61 19	1.59 64	3.35 8	2.06 64	4 0.95 11	1.05 1	9 1.21 8	1.16 51 1	1.25 54	4 1.58 59	1.22 68	1.85 70	1.12 56	0.85 47	1.73 49	0.70 18	0.96 45	1.43 4	0.64 34
Local-TV-L1 [65]	41.9	<u>0.65</u> 59	0.83 3	9 0.69 76	0.88 70	1.15	0.76 72	0.87 2	1.27	5 0.71 4	4 <u>1.01</u> 41	1.06 2	1 1.31 <mark>68</mark>	1.13 46	1.19 45	5 1.35 48	<u>1.14</u> 36	1.48 23	1.13 63	<u>0.81</u> 42	1.63 44	0.73 33	<u>0.85</u> 16	1.23 16	0.66 71
CRTflow [87]	42.0	<u>0.64</u> 54	0.89 5	8 0.67 <mark>62</mark>	0.83 57	1.16	se 0.69 se	1.12 32	1.66 3	2 1.02 2	B <u>1.00</u> 40	1.10 3	3 1.28 55	1.18 60 1	1.27 59	9 1.69 72	<u>1.05</u> 7	1.33 6	1.05 11	0.93 63	1.95 64	0.69 14	<u>0.94</u> 35	1.40 39	0.63 4
SCR [73]	43.2	0.61 16	0.78 1	7 0.64 3	0.66 16	0.901	16 0.61 1s	1.63 66	3.39 8	3 2.09 6	5 <u>0.97</u> 20	1.09 3	0 1.22 20	1.20 71	1.31 76	5 1.68 <mark>69</mark>	<u>1.14</u> 36	1.61 38	1.08 31	0.93 63	1.96 65	0.80 59	<u>1.06</u> 76	1.60 77	0.63 4
Fusion [6]	43.6	<u>0.64</u> 54	0.94 64	4 0.65 26	0.70 2	0.98	27 0.61 19	1.35 55	1.48 2	0 1.70 5 6	6 <u>1.06</u> 63	1.267	1 1.22 20	1.12 45	1.20 47	7 1.22 39	1.29 79	2.07 83	1.19 81	0.78 36	1.54 37	0.72 29	0.85 16	1.24 17	0.64 34
Classic+NL [31]	44.1	0.62 32	0.82 3	6 0.65 <u>26</u>	0.67 21	0.92	21 0.62 26	1.56 63	3.237	9 1.95 6 1	1 <u>0.97</u> 20	1.11 3	6 1.25 35	1.17 57	1.27 5	9 1.54 56	1.17 48	1.71 52	1.09 37	0.92 60	1.92 61	0.77 47	<u>1.00</u> 60	1.50 62	0.634
ACK-Prior [27]	45.1	<u>0.61</u> 16	0.83 3	9 0.64 3	0.69 27	0.98	27 0.60 12	2.41 81	1.84 4	43.188	2 1.02 51	1.12 4	5 1.27 48	1.22 83	1.32 77	7 1.72 88	<u>1.17</u> 48	1.64 45	1.12 56	0.79 38	1.54 37	0.78 51	0.83 13	1.19 13	0.65 59
FESL [74]	45.2	0.61 16	0.77 1	1 0.64 3	0.66 16	0.91	20 0.60 12	1.18 42	2.09 5	в <mark>1.11</mark> з4	4 <u>0.99</u> 38	1.10 3	3 1.27 48	1.21 78	1.32 77	7 1.69 72	1.20 60	1.81 <mark>6</mark> 9	1.12 56	0.90 59	1.87 59	0.74 37	<u>1.06</u> 76	1.60 77	0.64 34
Black & Anandan [4]	45.8	<u>0.68</u> 75	0.967	1 0.69 76	0.94 80	1.21	76 0.76 72	2.33 79	1.75 3	7 2.52 7	5 <u>1.08</u> 65	1.15 5	4 1.25 35	1.04 17	1.05 17	7 1.16 23	<u>1.11</u> 24	1.54 25	1.07 24	0.73 22	1.37 23	0.70 18	<u>0.87</u> 20	1.26 19	0.66 71
Classic+CPF [90]	46.2	<u>0.61</u> 16	0.79 2	2 0.64 3	0.68 25	0.93	24 0.62 26	1.21 43	2.25 6	2 1.22 3	<u>0.94</u>	1.02	8 1.19 1	1.22 83	1.34 84	4 1.69 72	1.23 72	1.88 75	1.11 51	0.92 60 [•]	1.92 61	0.74 37	<u>1.14</u> 90	1.73 91	0.65 59
NL-TV-NCC [25]	46.6	0.63 47	0.84 4	5 0.65 26	0.77 44	1.10	7 0.64 39	1.02 21	1.71 3	4 0.90 1	8 <u>1.07</u> 64	1.307	7 1.32 71	1.07 29 1	1.06 18	8 1.38 51	1.25 75	1.91 76	1.14 67	0.75 28	1.40 26	0.74 37	<u>0.99</u> 55	1.46 53	0.66 71
Adaptive [20]	46.7	0.64 54	0.91 6	0 0.66 47	0.88 70	1.22	79 0.71 63	1.06 27	1.76 3	9 1.05 3	0 <u>1.03</u> 52	1.17 5	9 1.33 75	1.09 36	1.12 36	6 1.15 <u>6</u>	1.20 60	1.78 63	1.14 67	0.86 50	1.76 52	0.71 24	<u>0.93</u> 33	1.38 34	0.63 4
Efficient-NL [60]	47.0	<u>0.61</u> 16	0.77 1	1 0.64 3	<u>0.71</u> 34	0.99	30 0.62 <u>2</u> 6	2.03 75	1.80 4	3 2.75 7	8 <u>0.97</u> 20	1.11 3	6 1.22 20	1.18 60 1	1.28 62	2 1.64 <mark>64</mark>	<u>1.19</u> 58	1.77 60	1.09 37	0.96 70	2.04 71	0.78 51	<u>1.10</u> 85	1.65 8	0.64 34
Occlusion-TV-L1 [63]	47.2	0.62 32	0.86 5	1 0.66 47	0.85 61	1.20	4 0.68 52	0.92 11	1.58 3	0.90 1	B 1.13 74	1.438	1 1.30 65	1.04 17	1.06 18	B 1.156	1.20 60	1.78 63	1.15 72	0.89 55	1.54 37	0.83 66	1.04 69	1.56 69	0.634

Fig. 7. Evaluation results on the Middlebury dataset. The proposed method AGIF+OF is highlighted. (a) The AAE. (b) The EPE. (c) The ANIE.

Table 8

AAE/EPE results of eight training sequences from the Middlebury dataset of methods: HS [1] and HS+AGIF, BA [2] and BA+AGIF.

	Urban2	Urban3	Grove2	Grove3	RubberWhale	Venus	Dimetrodon	Hydrangea
HS	4.060/0.459	7.520/0.856	2.853/0.204	6.809/0.690	3.798/0.118	5.533/0.337	4.675/0.224	2.218/0.190
HS+AGIF	2.999/0.282	5.749/0.668	1.975/0.143	5.638/0.550	3.026/0.096	3.621/0.245	4.363/0.208	2.047/0.174
BA	2.965/0.376	4.728/0.605	2.492/0.172	6.496/0.660	3.156/0.097	4.752/0.293	4.110/0.199	2.060/0.177
BA+AGIF	1.909/0.225	3.529/0.472	1.484/0.105	5.346/0.519	2.171/0.067	3.135/0.220	3.060/0.154	1.926/0.164

Evaluation results on the MPI-Sintel test set.

	AGIF+OF	MDP-Flow2 [2]	Classic+NL [10]
EPE (clean)	5.766	5.837	7.961
EPE (final)	8.514	8.445	9.153

errors which are caused by inaccurate flow vectors in the warped interpolation image $I_{\rm warp}$. This technique is especially suited to reduce errors due to outliers and occlusions. With this contribution, the accuracy of the discrete approximate temporal derivative

 I_t is modified. Consequently, this technique is useful for all Euler– Lagrange based variational methods. The AGIF technique is based on the GIF method but with two improvements. One improvement is that, we reconstruct a guidance image by properly combining the reference image I_1 and I_{warp} according to a temporal derivative measured error weight. As have shown, useful information (e.g., intensity and edge) will transferred from the reference image to the filtered I_{warp} . The other improvement is that, we use an error confidence to adaptively select the optimal regularization parameter ε , which can properly adjust the degree of smoothing. In this work, we did not find a sufficient way to use the GIF method to



Fig. 8. Visual comparison of the estimated flow fields on sequences Backyard, Basketball, and MiniCooper (each row, from left to right) from Middlebury dataset. (a), (d) and (g) Estimated flow fields of the method Classic+NL. (b), (e) and (h) Estimated flow fields of the method Classic+CPF. (c), (f) and (i) Estimated flow fields of the proposed method AGIF+OF.



Fig. 9. Visual comparison of the proposed AGIF+OF method and the MDP-Flow2 method [2] on the Football sequence. (a) Frame 3. (b) Frame 4. (c) The result of the AGIF+OF method. (d) The result the MDP-Flow2 method.

improve the accuracy of the spatial derivatives I_x and I_y , which will be a challenge for future work.

5.1. Limitations

There are several limitations of the AGIF+OF method. First, as shown in Tables 5 and 6, the improvement of AGIF over GIF is modest, because former is based on the latter and both have the same filtering process. Moreover, the approach to adaptively selecting the optimal regularization parameter ε is according to various experiments, we do not find a proper mathematical selection measure (Eq. (18)) which causes the selection measure to still suboptimal. In Table 5, we can see that the AGIF method is not better than the GIF method with respect to the Urban3.

Second, the method to combine I_1 and I_{warp} to reconstruct a guidance image G for AGIF cannot handle all variations. For different sequences, the error degree of I_{warp} is totally different. Even for one sequence, the error degree between different areas may have large differences. Creating a more appropriate combined approach is very important. Furthermore, splitting the reference image I_1 into different parts, capturing useful information and discarding invalid information should be done, but an effective way to do this has yet to be found. The results of the MPI-Sintel test set demonstrate the urgency of this important work: for the clean pass and the final pass, our method has different performances – better than MDP-Flow2 [2] in the clean pass while worse than MDP-Flow2 in the final pass. This is because the two passes have different complexities.



Fig. 10. Results of the proposed AGIF+OF method on real sequences 0000053 and 0000163 from the KITTI dataset. (a) The estimated flow field of the sequence 0000053. (b) The corresponding backward warped image of the sequence 0000053. (c) The estimated flow field of the sequence 0000163. (b) The corresponding backward warped image of the sequence 0000053.

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